OUR LADY OF AFRICA S.S NAMILYANGO (OLAN)
A LEVEL SUBSIDIARY MATHEMATICS S475 SEMINAR QUESTIONS 2022

PURE MATHEMATICS

ALGEBRA

1. (a) The second term of an A.P is 15, and the fifth is 21. Find the common difference, the first term and the sum of the first ten terms.

   (b) The three numbers $n-2, n, n+3$,are consecutive terms of a G.P. Find n, and the term after $n+3$.

   (c) A man deposits shs 800,000 into his savings account on which interest is 15% per annum. If he makes no withdrawals, after how many years will his balance exceed shs, 8 $millions$.

2. (a) Simplify (i) $\sqrt{8}+\sqrt{18}-2\sqrt{2}$ and (ii) $\sqrt{24}-3\sqrt{6}-\sqrt{216}+\sqrt{294}$.
   (b) Express $\frac{2}{3-\sqrt{2}}$ in the form $A+B\sqrt{C}$,

3. (a) The roots of the equation $x^2-px+8=0$ are α and $\alpha+2$. Find two possible values of p.

   (b) Prove that if the difference between the roots of the equation $ax^2+bx+c=0$ is 1, then $a^2=b^2-4ac$.

   (c) Find the values of a and b if $ax^4+bx^3-8x^2+6$ has a remainder of $2x+1$ when divided by x^2-1.

4. (a) Solve the following pair of simultaneous equations, $log_2^{(x+y)}=log^{100}$ and $log_2^{(2x-y)}=log^{10}$.

   (b) Solve by completing squares: i) $t^2-4t-8=0$ (ii) $2t^2-6t+4=0$

   (c) Solve for x , in $e^{3x}-2e^{2x}-e^{x}+2=0$

   (d) Express $f(x)=2+x-3x^2$ in the form $a-b(x+c)^2$, hence, deduce the maximum value of $f(x)$ and the value of x at which it occurs.

ANALYSIS

5. (a) Differentiate (i) $(x^2+3x)^7$, (ii) $(x+3)\sqrt{(1-x^3)}$, (iii) $\sqrt{\left(\frac{1+x^2}{x}\right)}$

   (iv) $y=3\sqrt[3]{x}+2\sqrt{x}$, (v) $y=3x^2cosx$.

   (b) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ when $x^2-3xy+y^2-2y+4x=0$.

(c) Find the turning points of the curve $y = 2x^3 + 3x^2 - 12x + 7$, distinguishing between maximum and minimum values and hence sketch the curve.
(d) A soap manufacturer can sell x bars of soap per week at p shillings each, where $5x = 375 - 3p$, and the cost of production is $500 + 5x + \frac{x^2}{5}$ shillings;

(i) Find how many bars of soap he should manufacture for maximum profit.

(ii) What will be the maximum profit.

6. (a) Integrate: (i) $\int(3\sqrt{x} - \sqrt[3]{x})\ dx$ (ii) $\int(\frac{x^2}{3} + \frac{3}{x^2})\ dx$ (iii) $\int \frac{x^2 sec^2 x + x^5}{x^2}\ dx$

(b) Evaluate: (i) $\int_0^6 2x(x^2 + 3)dx$ (ii) $\int_1^2 \frac{(x^2+1)^2}{x^2}\ dx$ (iii) $\int_0^{\frac{\pi}{3}} \left(\frac{3x^4 + 2x^2 cosx}{x^2}\right) dx$. (c) The gradient of a certain curve is kx. If the curve passes through the point (2,3) and the tangent at this point makes an angle of $tan^{-1}(6)$ with the positive direction of $x - axis$ find;

(i) value of k
(ii) The equation of the curve
(iii) The equation of the normal at the point (2,3)

(d) A wound heals so that the rate of decrease of its area A is $3\sqrt{t}\ mm^2$ per day where t is the time (in days) since the wound was inflicted. Given that the wound had an initial area of $16mm^2$.

(i). Find an expression for A in terms of t.
(ii). Determine how long the wound took to;

(a)Reduce to an area of $14mm^2$.
(b). Heal completely.

TRIGONOMETRY

7. (a) Find the reflex angle θ such that; $2sin^2\theta + cos\theta + 1 = 0$
(b) Solve the equation $2sin2\theta = 3cos\theta$, $for\ 0^o \le \theta \le 360^o$.

8. (a) Prove that: (i) $\frac{sin\theta + tan\theta}{1 + cos\theta} = tan\theta$ (ii) $\frac{1 + \sin x + \cos x}{1 + \sin x - \cos x} = \cot\frac{1}{2}x$

(iii) $\sin(x + y)\sin(x - y) = sin^2 x - sin^2 y$

(b) Eliminate $\theta$from the following equations equations

(i) $x = acos\theta$, $y = bsin\theta$ (ii) $x = sin\theta + cos\theta$, $y = sin\theta - cos\theta$

VECTORS

9. (a) Given that the position vector of A is $2i + 2j$ and $\overrightarrow{BA} = 7i - j$. Find;
(i) The position vector of point B. (ii) $\overrightarrow{OM}$, such that $\overrightarrow{AM} = \frac{1}{2}\overrightarrow{AB}$

(b) Given that $a = 3i - 4j, b = -5i + 12j$. Find i) $(3a + b).b$ ii) angle between a and b.
(c) Given that $a = -2i + 4j, b = -5i + 10j \; and \; c = 3i + 4j$;
$(i) \; Find \; |a + 2b - 3c|$
(ii) If r is a vector such that $r = c + \lambda(a - b)$ and $|r| = 10$, find the possible values of λ

MATRICES

10. (a)Given that matrices: $A = \begin{pmatrix} 5 & 1 \\ 0 & 2 \end{pmatrix}, B = \begin{pmatrix} -2 & 3 \\ 1 & 0 \end{pmatrix} \; and \quad C = \begin{pmatrix} 2 & 1 & -1 \\ 1 & 5 & 2 \end{pmatrix}$

Find: (i) ABC (ii) $(A + B)C$

(b) Given that A is the matrix $\begin{pmatrix} 2 & 4 \\ -1 & 3 \end{pmatrix}$;

(i) Determine the scalars x and y such that $A^2 + xA + yI = 0$, where I is 2x2 identity matrix.

(ii) Show that the inverse matrix of A can be expressed as $-\frac{1}{y}(A + xI)$. Hence find A^{-1}.

(c) By use of matrix method; solve the simultaneous equations below:
$3x + 4y = 8, \quad x + 2y = 3$

APPLIED MATHEMATICS

STATISTICS

11. A population consists of the 15 numbers 2, 4, 7, 3, 5, 6, 3, 6, 10, 7, 8, 9, 3, 4, 3.
Find: (i) the mode.(ii) the median (iii) the mean (iv) standard deviation

12. The heights and masses of ten students are given in the table below.

Heights (Xcm)	156	151	152	146	160	157	149	142	158	141
Mass (Ykg)	62	58	63	58	70	60	55	57	68	56

(i) Plot the data on a scatter diagram.
(ii) Draw the line of best fit and hence estimate the mass corresponding to a height of 155cm.
(iii) Calculate the rank correlation coefficient and comment about your results at 5% significance.

13. The table below shows the amount of money (in thousands of shillings) that was paid out as allowance to participants during a certain workshop.

Amount (sh'000s)	No. of participants
110 – 114	13
115 – 119	20
120 – 129	32
130 – 134	17
135 – 144	16
145 – 159	12

(a) Draw a histogram and use it to estimate the modal allowance.

(b) Calculate; (i) Median allowances. (ii) Mean allowances. (iii) Standard deviation

14. The table below summarizes the prices and quantities of items A, B, C traded by a given company in 2001 and 2008.

ITEMS	2001		2008	
	QTY	Price	QTY	Price
A	41.5	3.85	42.5	4.60
B	55.0	3.70	60.0	4..35
C	83.0	2.60	78.0	2.90

Calculate:

(i) Paasche's aggregate price index

(ii) Lespeyre's aggregate price index and comment on te general price index in 2008

(iii) value index

15. The average prices of a bunch of banana and a kilo of sugar in each quarter of a year are given in the table below.

	1st	2nd	3rd	4th
1998	4500	5000	5200	5500
1999	5500	5700	6000	6400
2000	6200	6500	6800	7200
2001	7400	Y		

(a) Calculate the 4 point moving averages.

(b) On the same graph, plot the raw data and the 4 point moving averages

(c) What is the purpose of plotting moving averages

(d) Hence comment on the trend of the prices for this period and use your graph to estimate the value of Y.

PROBABILITY

16. A random variable X has p.d.f, f(x) given as

X	1	2	3	4	5
P (X = x)	0.10	p	0.20	q	0.30

Given that E(x) = 3.5,

(i) Determine the values of p and q,
(ii) Calculate the standard deviation of x,
(iii) $Var\ (3 - 2x)$ (iv) Find P $(x \geq 2/x \leq 4)$

17. (a) Given that A and B are mutually exclusive events and P(A) = $\frac{2}{5}$ and $P(B) = \frac{1}{2}$, find: (i) P(AnB) ii) P(An$\bar{B}$) iii) P($\bar{A}$n$\bar{B}$)
(b) At a bus park, 60% of the buses are of Isuzu make, 25% are styer type and the rest are of Tata make. Of the Isuzu type, 50% have radios, while for the Styer and Tata types, only 5% and 1% have radio, respectively. If a bus is selected at random from the park, determine the probability that:
(i) it has a radio
(ii) a Styer type is selected given that it has a radio.

18. A random variable **X** has the probability density function:

$$f(x) = \begin{cases} kx; 0 < x < 1 \\ \left(\frac{k}{2}\right)x; 1 \leq x \leq 2 \\ 0; elsewhere \end{cases}$$

(a) Find; - (i) the value of k *(ii)* the expectation $E(x)$ (iii) the median of x
(b) Find the $P\left(\frac{1}{2} \leq x \leq 1\frac{1}{2}\right)$

19. The marks obtained by U.A.C.E candidates were found to be normally distributed with mean 50 and standard deviation 10.
(a) Determine the percentage of candidates who obtained; -
(i) more than 70 marks.

(ii) between 40 and 60 marks.

(b) Assuming that the total number of candidates were 10,000. Find the number of candidates who scored; -

(i) more than 65. (ii) less than 45.

20. (a) In a large group of people, its known that 10% have a hot breakfast, 20% have a hot lunch and 25% have a hot lunch and a hot breakfast. Find the probability that a person picked at random from this group;
   (i) has a hot breakfast and a hot lunch
   (ii) has a hot lunch, given that he had a hot breakfast

   (b) Two bags contain similar balls. Bag **A** contains 4 red and 3 white balls while bag **B** contains 3 red and 4 white balls. A bag is selected at random and a ball is drawn from it. Find the probability that a red ball is drawn.

MECHANICS

21. (a) The vertices of a quadrilateral ABCD are A (4, 0), B (14 ,11), C (0 ,6), D (-10, -5). Prove that the diagonals AC, BD bisect each other at right angles, and that the length of BD is four times that of AC.
   (b) Five forces of 1N, 3N, 5N, 6N and 7N act along the lines AB, CB, AD, DB and CD respectively of a triangle ABCD. The direction of the forces is given by the order of the letters. The 6N force makes an angle of 30^0 with DC. Taking AB as horizontal, find the magnitude and direction of the resultant.

22. (a) Two points A and B are 2cm apart, if a force of 50N is used to move a body from A to B in 15 s.
   (i) Find the work done.
   (ii) Calculate the rate at which the force is acting.
   (b) The point P is 5 metres vertically above point O. A body of mass 0.6 kg is projected from P vertically downwards with a speed of $3.5ms^{-1}$. Find the speed of the body when it reaches O.

23. A stone Q is projected vertically upwards with a speed of $5ms^{-1}$ from the top of a cliff 10m above the ground. At the same time another stone P is projected vertically upwards from the bottom of the cliff at $12ms^{-1}$. Calculate the
   (a) Vertical distance between the two stones when Q is at its maximum.
   (b) Velocity of P when the first stone is at its maximum.
   (c) Time that elapses and position just before the two stones collide.

END